

On the perfect lattice actions of abelian-projected SU(2) QCD *

S. Fujimoto, [†] S. Kato, M. Murata and T. Suzuki

Department of Physics, Kanazawa University Kanazawa 920-1192, Japan

We study the perfect lattice actions of abelian-projected SU(2) gluodynamics. Using the BKT and duality transformations on the lattice, an effective string model is derived from the **direction-dependent** quadratic monopole action, obtained numerically from SU(2) gluodynamics in maximally abelian gauge. The string tension and the restoration of continuum rotational invariance are investigated using strong coupling expansion of lattice string model **analytically**. We also found that the block spin transformation can be performed **analytically** for the quadratic monopole action.

1. INTRODUCTION

The infrared effective theory of QCD is important for the analytical understanding of hadron physics. Abelian monopoles which appear after abelian projection of QCD [1] seem to be relevant dynamical degrees of freedom for infrared region [2]. Shiba and Suzuki [3] derived the monopole action from vacuum configurations obtained in Monte-Carlo simulations extending the method developed by Swendsen.

We studied the renormalized monopole action performing block spin transformations up to $n = 8$ numerically, and saw that scaling for fixed b looks good[4]. If the effective action obtained here is very near to the perfect action, the physical quantities from it should reproduce the continuum rotational symmetry, although the action is formulated on the lattice. In order to show restoration of rotational invariance, the direction-dependence of the renormalized monopole action is very important. Practically, it is difficult to evaluate the string tension numerically using monopole action for infrared region, since Wilson loop operators follow a simple exponential curve and they become too small within the statistical noise. So we try to evaluate the Wilson loops using the string model corresponding to the renormalized monopole action.

2. STRING REPRESENTATION FROM

MONOPOLE ACTION

We derive here the lattice string model using BKT(Berezinskii -Kosterlitz-Thouless) transformation.

Let us start from the following direction-dependent monopole partition function;

$$\mathcal{Z} = \sum_{\substack{k_\mu(x)=-\infty \\ (\partial'_\mu k_\mu(x)=0)}}^{\infty} \exp \left\{ - \sum_{x,y} \sum_{\mu=1}^D k_\mu(x) \mathcal{D}(x,y;\hat{\mu}) k_\mu(y) \right\},$$

where $D = 4$ is space-time dimensionality and closed monopole currents $k_\mu(x)$ are defined on the dual lattice. Operator $\mathcal{D}(x,y;\mu)$ is composed of direction-independent part \mathcal{D}_1 and -dependent part \mathcal{D}_2 ;

$$\begin{aligned} \mathcal{D}(x,y;\hat{\mu}) &\equiv \mathcal{D}_1(x-y) + \mathcal{D}_2(x-y;\hat{\mu}), \\ \mathcal{D}_2(x,y;\hat{\mu}) &= g_1(b) \cdot \delta_{x,y} + g_2(b) \cdot \delta_{x+\hat{\mu},y} \\ &+ g_3(b) \cdot \sum_{\gamma \neq \mu} \delta_{x+\hat{\gamma},y} + \dots, \end{aligned}$$

where $\mathcal{D}_1 \gg |\mathcal{D}_2|$ and $g_2(b) \neq g_3(b)$ etc. .

$b(\beta, n) = na(\beta) = \sqrt{\tilde{\kappa}(\beta, n)/\kappa_{phys}}$ is the physical length in unit of the physical string tension κ_{phys} . The dimensionless string tension $\tilde{\kappa}$ is determined by the lattice Monte-Carlo simulation, $a(\beta)$ is the lattice spacing and n is the number of blocking steps. The couplings of the above monopole action are determined using the extended Swendsen method([3-4]) We have found that four and six point interactions are very small for low-energy region of QCD[4]. Hence

*presented by S. Kato

[†]E-mail address:kato@hep.s.kanazawa-u.ac.jp

we consider above only quadratic interactions for monopole currents for simplicity.

Using the transformation suggested in Ref. [5], this type of monopole action can be transformed into the string model;

$$\begin{aligned} Z &= \text{const.} \sum_{\tilde{\sigma}_{\mu\nu}(x)=-\infty}^{\infty} \left(\prod_x \delta_{\partial'_\mu \tilde{\sigma}_{\mu\nu}(x)=0} \right) \exp \{ \mathcal{S}_{STR} \} \\ \mathcal{S}_{STR} &= -\pi^2 \sum_{x,y} \sum_{\mu < \nu} \tilde{\sigma}_{\mu\nu}(x) \left(\frac{1}{\Delta \mathcal{D}_1} \right) (x-y) \tilde{\sigma}_{\mu\nu}(y) \\ &\quad - \frac{\pi^2}{4} \sum_{x,y} \sum_{\mu \neq \nu} \epsilon_{\mu\nu\xi\eta} \epsilon_{\mu\alpha\gamma\delta} \tilde{\sigma}_{\xi\eta}(x - \hat{\xi} - \hat{\eta}) \\ &\quad \times \left(\frac{\mathcal{D}_2}{(\Delta \mathcal{D}_1)^2} \right) (x-y; \hat{\mu}) \partial'_\alpha \partial_\nu \tilde{\sigma}_{\gamma\delta}(y - \hat{\gamma} - \hat{\delta}) \\ &\quad - (\text{higher order term}) \end{aligned}$$

where we used

$$\mathcal{D}^{-1} = \mathcal{D}_1^{-1} + \mathcal{D}_1^{-1} \mathcal{D}_2 \mathcal{D}_1^{-1} + \dots$$

The integer-valued plaquette field $\tilde{\sigma}_{\mu\nu}(x)$ which is defined on the original lattice represents the closed world surface formed by a color electric flux tube. The leading part of this model comes from the direction-independent part of the monopole action; the next-leading terms come from the contribution of the direction-dependent part.

3. ROTATIONAL INVARIANCE

In order to check the restoration of continuum rotational symmetry, let us consider the $q\bar{q}$ static potential at the points $(2,0,0)$ and $(1,1,0)$ of a three-dimensional time-slice, respectively. (The quark is attached on the origin $(0,0,0)$ and anti-quarks are attached on $(2,0,0)$ and $(1,1,0)$, respectively.)

The static potential $V(x,y,z)$ are calculated from the Wilson loop operators:

$$\begin{aligned} V(2,0,0) &= - \lim_{T \rightarrow \infty} \frac{1}{T} \log \langle W(2,0,0,T) \rangle \\ V(1,1,0) &= - \lim_{T \rightarrow \infty} \frac{1}{T} \log \langle W(1,1,0,T) \rangle. \end{aligned}$$

If the potential is purely linear and the continuum rotational symmetry is restored, then the ratio $V(2,0,0)/V(1,1,0)$ should become $\sqrt{2}$.

The quantum average of Wilson loop operator in the string representation is written as follows.

$$\langle W(C) \rangle = \frac{1}{Z_{STR}} \sum_{\substack{\tilde{\sigma}_{\mu\nu}(x)=-\infty \\ (\partial'_\mu \tilde{\sigma}_{\mu\nu}(x)=\tilde{J}_\nu(x))}}^{\infty} \exp \{ \mathcal{S}_{STR}[\tilde{\sigma}_{\mu\nu}(x)] \}$$

Note that the string field $\tilde{\sigma}_{\mu\nu}(z)$ are forming open surfaces whose boundaries are the Wilson loop. We can evaluate this quantity using **strong coupling expansion**.

At $b = 2.14$ ($\cong 0.96$ fm), as a preliminary result, the string tension from $V(2,0,0)$ become 1.5 in unit of κ_{phys} and the ratio $V(2,0,0)/V(1,1,0) = 1.07$. The quadratic part of the renormalized direction-dependent monopole action does not seem to reproduce the correct string tension and the continuum rotational invariance. We probably need (1) to consider 4 and 6 point interactions, (2) more steps of blocking on larger lattice volume for larger β and (3) more complicated form of monopole action.

4. ANALYTIC BLOCK SPIN TRANSFORMATION

We found that the block spin transformation can be performed **analytically** for the quadratic monopole action.

When b is large, the London limit of the abelian Higgs model works good as an effective theory of QCD. For example, let us start from the following simple monopole partition function defined on the small $a(\beta)$ -lattice:

$$\begin{aligned} Z &= \sum_{\substack{a^3 k=-\infty \\ (\partial'_\mu k_\mu(x)=0)}}^{\infty} \exp \left\{ - \sum_{s,s';\mu} k_\mu(s) D(s-s') k_\mu(s') \right\} \\ D(p) &= \left(\alpha + \frac{\beta}{4 \sum_\rho \sin^2(\frac{p_\rho}{2})} \right) \left(1 + 4\epsilon \sum_\rho \sin^2(\frac{p_\rho}{2}) \right) \end{aligned}$$

This action corresponds to the London limit of the abelian-Higgs model, when $\epsilon = 0$.

Using the Poisson summation formula, the monopole action defined on the $b = na(\beta)$ lattice is given by

$$e^{-S[K]} = \sum_{a^3 k_\mu(a s)=-\infty}^{\infty} \delta \left(\sum_\mu \partial'_\mu k_\mu(a s) \right) \delta \left(b^3 K_\mu(b s) - \mathcal{M} \right)$$

$$\begin{aligned}
& \exp \left\{ - (a^4)^2 \sum_{s,s';\mu} k_\mu(as) D(as - as') k_\mu(as') \right\} \\
& \text{where,} \\
& \mathcal{M} \equiv \sum_{i,j,l=0}^{n-1} a^3 k_\mu \left(nas + (n-1)a\mu + ia\nu + ja\rho + la\sigma \right), \\
& S[K] = \int_{-\pi/na}^{+\pi/na} \frac{d^4 p}{(2\pi)^4} K_\mu(-p) [\Delta_{\mu\nu}^{GF}(p)]^{-1} K_\nu(p) \\
& \text{and} \\
& \Delta_{\mu\nu}(p) = \frac{1}{(\alpha - \beta\epsilon)} \{ \bar{\Delta}_{\mu\nu}(p, \epsilon^{-1}) - \bar{\Delta}_{\mu\nu}(p, \frac{\beta}{\alpha}) \} \\
& \bar{\Delta}_{\mu\nu}(p, m^2) \equiv \frac{m^2}{n^6} \sum_{l=0}^{n-1} \frac{\Pi_{-\mu}(p + 2\pi l) \Pi_{-\nu}(p + 2\pi l)}{4 \sum_{\rho} \sin^2(\frac{p_\rho a}{2} + \frac{\pi l_\rho}{n}) + m^2} \\
& \times \left(\delta_{\mu\nu} - \frac{\sin(p_\mu a + \frac{\pi l_\mu}{n}) \sin(p_\nu a + \frac{\pi l_\nu}{n})}{\sum_{\rho} \sin^2(p_\rho a + \frac{\pi l_\rho}{n})} \right) e^{\frac{i}{2}(p_\mu - p_\nu)na} \\
& \Pi_{-\mu}(p) \equiv \prod_{i \neq \mu} \frac{\sin(p_i na/2)}{\sin(p_i a/2)}.
\end{aligned}$$

Using the above analytical block spin transformation, we can find a lattice monopole action which reproduces the continuum rotational invariance with the correct string tension if we take $n \rightarrow \infty$ and $a(\beta) \rightarrow 0$ for fixed $b = n \cdot a(\beta)$ [6].

Let us evaluate the string tension and the ratio of the potential from string model expression corresponding to $S[K]$ above. The ratio $V(2, 0, 0)/V(1, 1, 0) = 1.24$ at $b = 2.14$ ($\cong 0.96$ fm).[‡] It approaches considerably to the value expected from the continuum rotational invariance. The discrepancy may be due to truncation effect of the monopole action.

5. CONCLUSIONS

Let us compare the monopole action $S[k]$ in previous section with the present numerical data fixed by the Swendsen method. We see the self coupling G_1 and the discrepancy between nearest-neighbor couplings (G_2 and G_3) are larger than those of the monopole action determined numerically as shown in Figure 1. These behavior must be important to reproduces the continuum rotational invariance.

Recently, the spectrum of glueball masses in non-supersymmetric Yang-Mills theory based on

[‡]The parameters $\alpha = 0.73, \beta = 0.73$ and $\epsilon = 0.01$ are determined so that the string tension becomes unity in unit of physical one at $b = 2.14$.

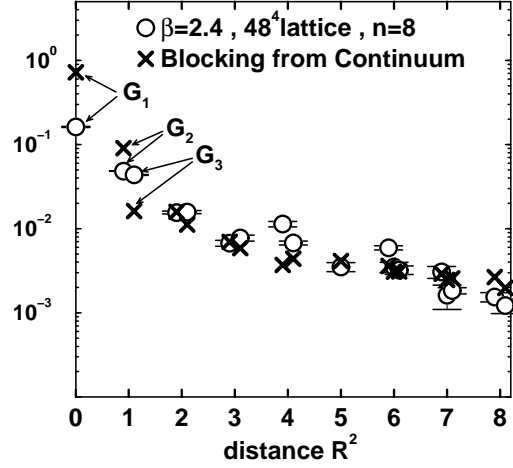


Figure 1. The R^2 dependence of the monopole action from Swendsen's method and analytical block spin transformation at $b = 2.14$.

a Maldacena's conjectured duality between supergravity and large N gauge theories are evaluated[7]. Our string model also yields glueball mass spectrum analytically using the strong coupling expansion of the correlation functions of gauge invariant local operators. This is now in progress.

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